TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. (4)

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta = 1-\frac{1}{\sqrt{2}}, (k>0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. (1)

3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

4. For each xεR, let [x] be the greatest integer less than or equal to x. Then

$\lim_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0 (3) 1 (4) sin1

Ans. (1)

5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4) (3) (5,6) (4) (–5,–4)

Ans. (Bonus)

6. If

$$A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Then A is-

(1) Invertible only if $t = \frac{\pi}{2}$

(2) not invertible for any tεR

(3) invertible for all tεR

(4) invertible only if t=π

Ans. (3)

7. The area of the region

$A = [(x,y): 0 \le y \le x|x|+1 \text{ and } -1 \le x \le 1]$

in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3+6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

9. Let $\vec{a} = \hat{i}+\hat{j}+\sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

10. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

22. If $f(x)=\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \ge 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

23. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x-4)^2 + (y-7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$
(3) $r > 11$ (4) $r = 11$

Ans. (2)

24. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is :

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

25. The sum of the follwing series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}$$

$$+\frac{15(1^2+2^2+....+5^2)}{11}+....\text{ up to 15 terms, is:}$$

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

26. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

(1) $\frac{1}{2}$ (2) 4 (3) 2 (4) $\frac{7}{13}$

Ans. (2)

27. If the system of linear equations

$x-4y+7z = g$
$3y - 5z = h$
$-2x + 5y - 9z = k$

is consistent, then :

(1) (
(2) $2g + h + k = 0$
(3) $g + h + 2k = 0$
(4) $g + 2h + k = 0$

Ans. (2)

28. Let $f:[0,1]\to R$ be such that $f(xy) = f(x).f(y)$ for all $x,y,\varepsilon[0,1]$, and $f(0)\neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

29. A data consists of n observations:

$x_1, x_2, \ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2=9n$ and $\sum_{i=1}^{n}(x_i-1)^2=5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)